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μ Synthesis for a Small Commercial Aircraft: Design and Simulator Validation

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The potential benefits deriving from the application of modern multivariable techniques to the flight control laws design for a fly-by-wire small commercial aircraft are evaluated by means of a practical benchmark problem. For such a class of airplanes, a unified set of flying qualities requirements is not available; therefore, a first effort has been made to formulate a suitable set of requirements to be followed during the design phase. On the basis of such requirements, a reference dynamic model of the aircraft has been chosen. Then a model matching H_∞ control problem has been solved via the μ synthesis approach, which allows disturbance rejection and robustness specifications to be taken into account. Finally, the designed flight control laws have been evaluated by means of both numerical off-line simulations and pilot-in-the-loop simulations performed via the ground-based simulator located at the National Aerospace Laboratory in Amsterdam. The research activity has been performed within the framework of the research project Affordable Digital Fly-By-Wire Flight Control Systems funded by the IV Framework Program (1997–2000) of the European community.

I. Introduction

DURING the last decades, several applications of full-authority fly-by-wire (FBW) flight control systems to civil aircraft have been experienced. In FBW systems, the commands for the actuators of the control surfaces are computed by the onboard flight control computer based on pilot activities and without any mechanical linkage. This allows a significant reduction of pilot workload and results in a general improvement of the flying qualities (FQ) characteristics. However, the high development costs of a FBW flight control system historically has made it affordable only for large commercial aircraft.

Moreover, for most FBW aircraft flying today, the control laws were developed by the use of classical single-loop frequency responses and root locus design techniques. This approach to aircraft control design is essentially the same as that outlined in Ref. 1. Over the past 30 years, new multivariable control laws analysis and synthesis techniques have been proposed. These techniques have their roots in the theories of the optimal process by Bellman[2] and Pontryagin et al.[3] Subsequently, many extensions and variations have been investigated by the academic community, whose main products are the H_∞ theory and the related μ synthesis approach. (See Refs. 4–7 for theoretical developments and Refs. 8–12 for some meaningful applications.) The proponents of multivariable control theory claim that the new techniques can handle multiloop control problems in a formal and systematic manner.

On the other hand, the main limitations of classical single input/single output (SISO) methods are the considerable time consumed for the control system design and tuning, the suboptimal performance due to the increasing coupling among control channels, and the poor robustness to the possible model uncertainty sources (Refs. 4, 5, and 8). However, despite the availability of good computational algorithms and software, practicing control engineers in industry have been reluctant to adopt and use the new techniques.

Therefore, only in 1978 did The Boeing Company start the first significant application of the multivariable control methodologies. The work was sponsored by NASA and performed under a project entitled Integrated Application of Active Control Technology to an Advanced Subsonic Transport. The results clearly demonstrated that multivariable control laws design techniques offered significant advantages over classical techniques in the solution of multiloop control problems. Command and stability augmentation, gust load reduction, and flutter suppression control functions were successfully implemented by using the linear quadratic (LQ) optimal control approach.[13,14] (Also see Refs. 15 and 16 for further applications of LQ theory in the aerospace context.) The outcome of a companion activity adopting classical single-loop techniques was the design of a control system offering significantly less performance and robustness.

Motivated by the initial success, practical multivariable design methodologies have been further developed at Boeing and successfully applied to a wide range of control problems over the past 20 years as described in Ref. 15. Moreover, at NASA John H. Glenn Research Center at Lewis Field, a specific project aimed to the application of advanced multivariable control synthesis techniques in the aerospace industry was carried out.[17] In particular, in Ref. 8 details are provided of the H_∞ based multivariable control design for a short takeoff and vertical landing aircraft. For a survey of the key papers in robust multivariable control applied to aerospace systems, the interested reader is referred to Bates and Postlethwaite.[18]

To reduce the gap between theory and practice with respect to the application of modern control design techniques, a Group for Aeronautical Research and Technology in Europe (GARTEUR) action group (1994–1997), which involved aircraft manufacturers, research centers, and universities, was established to develop dedicated research activities. Its aim was to demonstrate, by means of design

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and off-line computer simulations, how 12 advanced methodologies could be applied to design robust controllers for two fairly realistic flight control benchmark problems.[19] To some extent, this activity proved that modern techniques could be used to design controllers for realistic applications. Additionally, it confirmed that those requirements for industrial application of new techniques are quite severe. Even if the presented methods had much potential in terms of improved robustness, better performance, decoupled control, and simplification of the design process, some methods did not yet have the maturity required for industrialization. In particular, the result of the project was that the main issues to be solved for future industrialization are the development of an efficient method for gain scheduling of multivariable controllers and the translation of the classical FQ requirements into the modern multi-input/multi-output (MIMO) control laws design process.

The activity described in Ref. 19 has continued through a new GARTEUR project, which deals more specifically with the problem of the clearance of the flight control laws.[20] However, the project ended very recently (September 2002), and the conclusions about the results of the research are currently under development.

The objective of the work described in this paper, carried out in the framework of the affordable digital FBW flight control system (Affordable Digital Fly-by-Wire Control System [ADFCS])[21] research project funded by the European community within the IV Framework Program (1997–2000), is the development, in the context of a practical benchmark problem, of a longitudinal stability and control augmentation system (SCAS) designed via a MIMO robust control synthesis technique. The proposed SCAS guarantees the optimal model matching of a suitable reference model, rejection of external disturbances, and robustness vs model uncertainties in a wide region of the aircraft flight envelope. This last point renders the scheduling phase much easier to accomplish because it is necessary to schedule among a relatively low number of different controllers.

The benchmark problem under consideration concerns a small commercial aircraft (SCA), whereas, as said, most of the previous FBW applications involved large commercial aircraft. For this reason, a big effort during the ADFCS project has been made to formulate a suitable set of FQ requirements for the SCA; a further contribution of this paper consists of incorporating such requirements into the control laws multivariable design process by the choice of a reference dynamic model of the aircraft that satisfies the level 1 FQ requirements.

The proposed MIMO approach makes use of the μ analysis and synthesis theory.[5–7] This approach is particularly suitable for aircraft control applications because it makes it possible to formulate and to solve numerically a design control problem that simultaneously takes into account performance requirements (in terms of model matching specifications and external disturbance rejection) and robustness requirements with respect to open-loop model uncertainties.

A numerical design is performed with reference to the benchmark model of the SCA, namely, a model of a twin jet powered aircraft with maximum takeoff weight of about 40,000 lb, a maximum altitude of 43,000 ft, and a maximum operating Mach of 0.87. The achieved results with the μ approach are compared, by means of off-line numerical simulations of the augmented aircraft model, with those ones achieved by using a classical design methodology based on the SISO approach.

Moreover, the proposed design approach is demonstrated also by means of numerical off-line models of the augmented aircraft, as well as by means of pilot-in-the-loop simulations performed at the flight simulator of the National Research Laboratory (NLR) in Amsterdam. The paper is organized as follows: In Sec. II, the benchmark control problem considered in the ADFCS project is described. In Sec. III, the proposed flight control laws (FCL) design process and the motivation for using the modern MIMO design techniques are detailed, by emphasizing the main differences between modern and classical design cycle. In Sec. IV, the robust SCAS design via the μ approach is described. Finally, in Secs. V and VI, the results achieved by means of off-line numerical analysis and pilot-in-the-loop simulations, respectively, are summarized.

II. Benchmark Control Problem

The considered benchmark flight control problem consists of designing the longitudinal SCAS of the FBW SCA whose main characteristics are 1) twin jet powered, 2) maximum takeoff weight of 42,000 lb, 3) maximum altitude of 43,000 ft, 4) maximum operating Mach at 0.87, and 5) minimum control speed of 100 kn calibrated.

The specific goal is to design the SCAS for the approach and landing flight envelope shown in Fig. 1. In particular, in the selected configuration flaps and slats are deployed and the landing gear is down. Moreover, a wide range of flight conditions is covered with calibrated airspeed (V_{cas}) from about 120 until 200 kn.

Successful design of control laws hinges on the availability of a set of quantitative design requirements or performance critera, which, in the case of an aircraft control system, can be very difficult to obtain. In particular, whereas in the military field some quantitative specifications have been defined and are currently used,[22] a formal set of criteria to be followed during the design phase of the FCLs for a civil FBW aircraft does not exist.

The first conceptual step of the approach proposed here consists of approximating the complex high-order frequency response dynamics of the augmented aircraft by a low-order system containing a time delay, the so-called low-order equivalent system (LOES).

Then some military criteria defined in Ref. 22 have been adapted to the SCA we deal with; these criteria establish a correlation between the pilot rating, that is, the FQ level, and some regions in the parameter space to which the LOES parameters, for example, short-period and phugoid frequencies and damping ratios) should belong. For example, the phugoid damping ratio criterion states that the damping of the phugoid mode has to be greater than 0.5 whatever the flight condition is. The FQ criteria based on the described approach are known as parametric.

In addition, further criteria based on the nonparametric graphical representation proposed by Gibson in Ref. 23 have been considered for the FCL design. These criteria define some limits on the shape of the responses both in the time and in the frequency domain without any reference to numerical mode parameters or mathematical formulas. They can be applied to any controlled response independently of the order and type of model available. For example, the pitch short-period main characteristics in the time domain, which appears to be of primary importance to predict the FQ, are the pitch rate overshoot and the time to first peak.

Briefly the selected parametric FQ criteria are[22] the control anticipation parameter (CAP), the phugoid damping ratio, and the equivalent time delay. Conversely, the criteria belonging to the

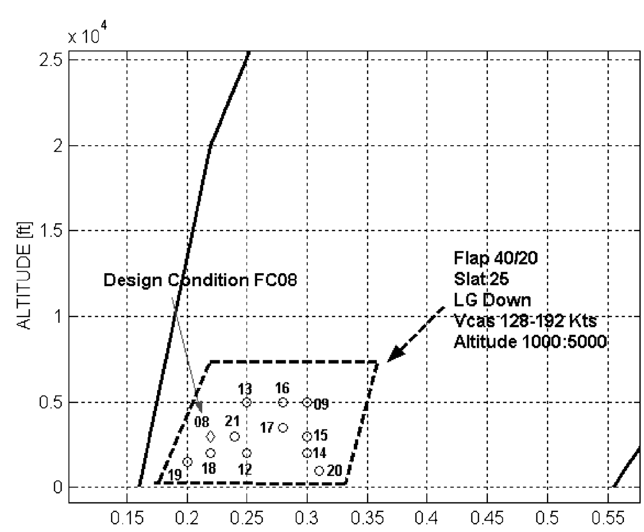

Fig. 1 Approach and landing flight envelope region.

nonparametric graphical category are[23] the dropback, the pitch rate time history, and the pitch attitude bandwidth.

III. FCL Design: Motivation for the Synthesis via Multivariable Techniques

The majority of current aircraft control systems are based on classical SISO techniques. A fundamental prerequisite to the application of such methods is an adequate a priori knowledge of the operational and behavioral characteristics of the aircraft so that the control system designer can carry out the synthesis process interactively.

Classical methods usually act on a multiloop architecture (Fig. 2) consisting of inner loops, which generally augment the stability [stability augmentation system (SAS)] and give the desired response type to the pilot commands by using also adequate feedforward control actions [control augmentation system (CAS)] and outer loops, which generally control the aircraft flight path (autopilot) and guarantee the required carefree handling (envelope protection modules). Because classical controller tuning techniques deal only with SISO systems, a very time-consuming one-loop-at-time design process must be necessarily used.

A common drawback of classical and modern control techniques derives from that they are mostly based on linear control theory, and hence, a gain scheduling is inevitably required to account for changes in operating points of the system. Nevertheless, multivariable robust control approaches allow designing robust controllers, which can operate over a wide region of the flight envelope, thus, reducing the number of different controllers to design and schedule.

Finally, for the robustness of the designed controller with respect to model uncertainties, the classical control methodologies provide theoretical results that may guarantee only the robust stability of the closed-loop system. This means that, to verify that performances are guaranteed in face of model uncertainties, a very time-consuming linear analysis of the augmented aircraft in various operating points must be performed.

On the other hand, the proposed multivariable robust control approach allows the design of the feedback and of the feedforward command shaping compensator to be addressed simultaneously, which in the classical approach is done in two iterative steps. When the robust FCL conceptual architecture shown in Fig. 3 is compared with the earlier mentioned classical one, it is clear that the multiloop SAS/CAS (SCAS) controller is replaced by a single MIMO controller.

The MIMO SCAS controller is obtained by solving a robust model-matching problem, which consists of designing the controller

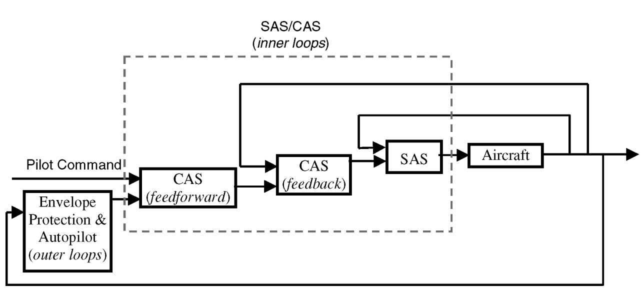

Fig. 2 Classical FCL scheme.

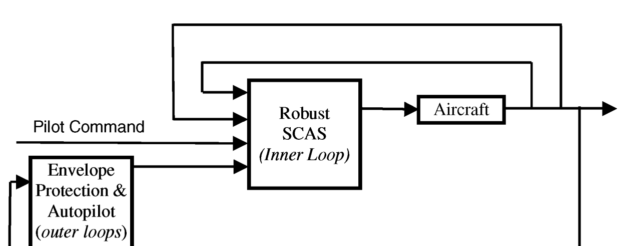

Fig. 3 Robust FCL scheme.

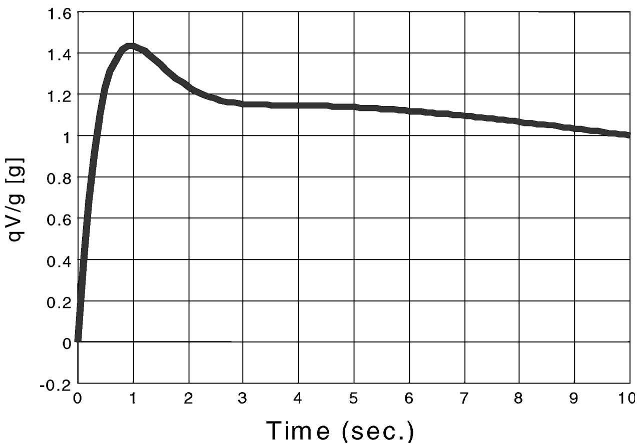

Fig. 4 Step response of the reference model.

such that the closed-loop system would be as close as possible to a selected reference dynamic model. This model is chosen to meet all level 1 FQ requirements defined in Sec. II simultaneously. The reference model is then the classical fourth-order transfer function between pilot stick force F_s and aircraft pitch rate q,

$$\frac{q}{F_s} = \frac{K_q s\,(1 + sT_{\vartheta 1})\,(1 + sT_{\vartheta 2})}{\left(s^2 + 2\xi_{\mathrm{ph}}\omega_{\mathrm{ph}} s + \omega_{\mathrm{ph}}^2\right)\left(s^2 + 2\xi_{\mathrm{sp}}\omega_{\mathrm{sp}} s + \omega_{\mathrm{sp}}^2\right)}$$

where for phugoid poles

$$\omega_{\mathrm{ph}} = 0.09 \text{ rad/s}, \qquad \xi_{\mathrm{ph}} = 0.9$$

and for short-period poles

$$\omega_{\mathrm{sp}} = 2 \div 3 \text{ rad/s}, \qquad \xi_{\mathrm{sp}} = 0.8, \qquad T_{\vartheta 1} = 5 \text{ s}, \qquad T_{\vartheta 2} = 1.1 \text{ s}$$

The pitch rate step response of the selected reference model is shown in Fig. 4. In this way it is possible to translate the FQ requirements into FCL specifications by the elimination of the very time-consuming iterative process needed in the classical design procedure to meet all design criteria simultaneously.

The design of the controller is achieved by using an approach, described in the next section, based on the μ synthesis theory. This design technique addresses the problem of retaining a desired performance level of the augmented system in the face of uncertainties, which is called the robust performance problem. The controller, in the form of a state-space linear model, is the result of the numerical optimization of a cost function, which simultaneously takes into account 1) performance specifications in terms of desired response type and related dynamic parameters of the augmented aircraft, 2) rejection of external disturbances such as sensor noise and wind gust, 3) robustness with respect to model parameter uncertainties, and 4) specification of control activity limitation, expressed in terms of maximum deflection and deflection rate of the control surfaces.

It will be shown that, by applying the robust control approach based on μ synthesis, we will achieve a fixed gain controller optimized in correspondence with a single design flight condition but working properly in a large portion of the flight envelope, whereas, as mentioned, by the use of classical design techniques heavy scheduling of the controller parameters is required.

To test the performance achieved by the proposed controller with the NLR flight simulator by means of pilot-in-the-loop simulations, the robust longitudinal SCAS has been integrated with the available classical lateral FCL and longitudinal envelope protection modules, developed within the research project of Ref. 21. This allowed the clear evaluation of possible benefits and weaknesses of the proposed control design technique and the definition of recommendations for future industrial applications.

The integration with classical modules has been performed saving, as much as possible, the classical FCL architecture, and only a retuning the related parameters has been done. Specifically, as shown in Fig. 5, the white blocks are used to describe the modules inherited from the classical FCL, the gray blocks have the same structure as the classical ones but their parameters have been retuned to cope with the new robust longitudinal SCAS, and the dotted block represents the robust SCAS system. Finally, note that the attitude-hold modules uses a proportional/integral controller architecture to guarantee the required zero steady-state error. Conversely, the classical attitude-hold block is a simply proportional controller because the integrator is already included in the inner SAS loop.

IV. Robust SCAS Design for a SCA

A. Preliminary Definitions and Notation

In this section, a short background on μ theory, as a framework for robust stability analysis and design, is provided. The general classification of uncertainty is between parametric uncertainties and unmodeled dynamics. All of these uncertainties can be "pulled out" from inside the plant as shown in Fig. 6, where the standard representation of a control system, subject to noise, disturbance and uncertainty, is reported.[5]

$P(s)$ is the nominal plant, $K(s)$ is the controller, and Δ is the set of all possible uncertainties, grouped into a single block-diagonal finite-dimensional linear time invariant system. For simplicity, we assume that the uncertainty block Δ is square, that is, $n_v = n_\eta = m$, where n_v and n_η are the dimensions of vectors $\boldsymbol{v}$ and $\boldsymbol{\eta}$, respectively. (The theory we shall provide in the sequel is entirely valid in the nonsquare case.) Moreover, $\boldsymbol{w}$ is the vector of plant inputs including noises, disturbances, and reference signals; $\boldsymbol{z}$ is the vector of plant outputs including the controlled variables and tracking errors; $\boldsymbol{u}$ is the control signal; and $\boldsymbol{y}$ is the measured output. To define the set to

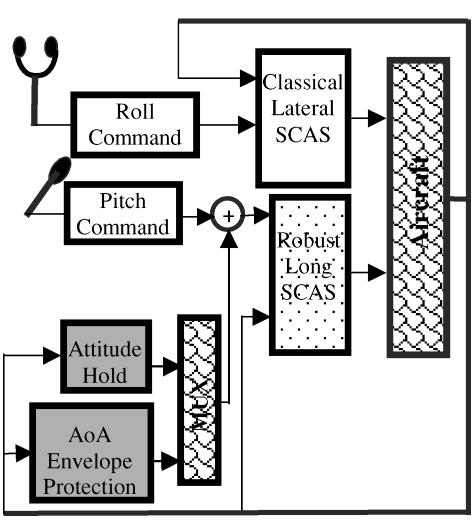

Fig. 5 Robust FCL architecture.

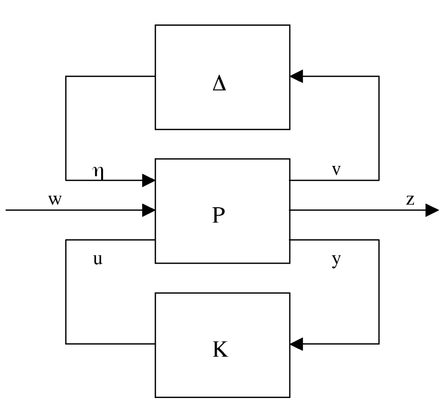

Fig. 6 Standard representation of control systems.

which the uncertain block Δ belongs, let m_r, m_c, and m_C be three integer numbers and consider the m-tuple $(m := m_r + m_c + m_C)$ of positive integers,

$$\boldsymbol{S} = (s_1, \ldots, s_{m_r}, s_{m_r+1}, \ldots, s_{m_r+m_c}, s_{m_r+m_c+1}, \ldots, s_m) \quad (1)$$

In the sequel we shall refer to $\boldsymbol{S}$ as the uncertainty structure. In correspondence to the structure $\boldsymbol{S}$, consider the matrix set

$$\Delta_S := \Big\{\Delta = \text{block diag}\Big(\delta_1^r I_{s_1}, \ldots, \delta_{m_r}^r I_{s_{m_r}}, \delta_1^c I_{s_{m_r+1}}, \ldots, \delta_{m_c}^c I_{s_{m_r+m_c}}, \Delta_1^C, \ldots, \Delta_{m_C}^C\Big)\Big\}$$

where $\delta_i^r \in R$, $\delta_i^c \in C$, and $\Delta_i^C \in C^{s_{m_r+m_c+i} \times s_{m_r+m_c+i}}$. In other words, the set Δ_S is composed of all block-diagonal matrices in which the first m_r blocks are identity matrices multiplied by the real scalars δ_i^r, the following m_c blocks correspond to identity matrices multiplied by the complex scalars δ_i^c, and the last m_C blocks are full complex blocks.

Now, given a matrix F of compatible dimensions, the structured singular value of F with respect to the block structure $\boldsymbol{S}$ is defined as follows:

$$\mu_S(F) := \begin{cases} 0 \quad \text{if} \quad \det[I - F\Delta] \neq 0 \quad \forall \Delta \in \Delta_S \\ \dfrac{1}{\left\{\min\limits_{\Delta \in \Delta_S} (\bar{\sigma}(\Delta)) : \det[I - F\Delta] = 0\right\}} \end{cases} \quad (2)$$

Next let us consider the uncertainty set

$$U(\Delta_S) := \{\Delta(s){:}\Delta(s) \quad \text{is stable and} \quad \Delta(j\omega) \in \Delta_S \forall \omega \in R, \|\Delta\|_\infty < 1\}$$

where $\|\Delta\|_\infty := \sup_\omega \bar{\sigma}[\Delta(j\omega)]$ and $\bar{\sigma}(\cdot)$ is the maximum singular value of the argument. Therefore, the set $U(\Delta_S)$ is composed of all stable block-diagonal transfer functions in which the first m_r blocks are related to real parametric uncertainties, the following m_c blocks correspond to complex parametric uncertainties, and the last m_C blocks are full complex dynamic uncertainties. Note that Δ_S is a matrix set, whereas $U(\Delta_S)$ is a transfer function set.

We can combine the plant $P(s)$and the compensator $K(s)$ of Fig. 6 into a single system $M(s)$, which is assumed to be stable, as shown in Fig. 7. By partitioning the compensated plant $M(s)$, we can write

$$\begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{z} \end{bmatrix} = M(s) \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{w} \end{bmatrix} = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{w} \end{bmatrix}; \boldsymbol{\eta} = \Delta v \quad (3)$$

If there exists a unique solution to the loop equation (3), we can rearrange the system in Fig. 7 into a unique mapping from the input $\boldsymbol{w}$ to the output $\boldsymbol{z}$,

$$\boldsymbol{z} = F_U(M, \Delta)\boldsymbol{w}$$

$$F_U(M, \Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1} M_{12}$$

where $F_U(M, \Delta)$ is the standard upper linear fractional transformation (LFT)[5] M-Δ of Fig. 7.

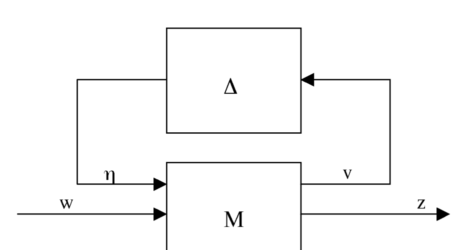

Fig. 7 Standard representation of control systems with combined plant and controller.

It is possible to show that an admissible perturbation Δ destabilizes system (3) if and only if

$$\det[I - M_{11}(j\omega)\,\Delta(j\omega)] = 0$$

for some ω and some $\Delta(\cdot) \in U(\Delta_S)$. From this last observation and definition (2) of the structured singular value the following theorem follows.

Theorem 1 (robust stability)[5]: The system shown in Fig. 7 remains stable for all $\Delta(\cdot) \in U(\Delta_S)$ if and only if

$$\sup_{\omega} \mu_S[M_{11}(j\omega)] \le 1$$

□

The μ framework also allows evaluating system robust performance. When exogenous disturbances act on the systems (wind gusts, sensor noise, etc.), the closed-loop performance can drastically be degraded. We say that the closed-loop system exhibits robust performance if the performance objectives are satisfied for all modifications of the plant due to uncertainties.

Assume $M(s)$ is a stable, real-rational, proper transfer function, with $n_w + n_\eta$ inputs and $n_z + n_v$ outputs, where $n_w = n_z = r$ and n_w and n_z are the number of exogenous inputs and the number of controlled variables, respectively. (Again we assume $n_w = n_z$ for the sake of simplicity. As before, the theory also holds in the general nonsquare case.) Define the augmented block structure as follows:

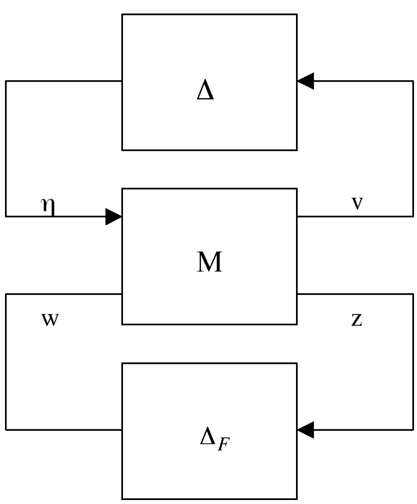

Fig. 8 Control systems with fictitious augmented uncertainty.

$$\Delta_P := \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_F \end{bmatrix} : \Delta \in \Delta_S,\, \Delta_F \in \Delta_{S_F} \right\}$$

where $\boldsymbol{S}_F$ is a block structure related to the block Δ_F defined in a way similar to Eq. (1), but is composed only of full complex blocks. Moreover, we denote the overall structure associated to the augmented block Δ_P by $\boldsymbol{S}_P$. We define the augmented uncertainty set

$$U\left(\Delta_{S_P}\right) := \left\{ \Delta_P(s) = \begin{bmatrix} \Delta(s) & 0 \\ 0 & \Delta_F(s) \end{bmatrix} : \Delta(s),\, \Delta_F(s) \text{ are stable}, \right.$$

$$\left. \Delta(j\omega) \in \Delta_S,\, \Delta_F(j\omega) \in \Delta_{S_F},\, \forall \omega \in R,\, \|\Delta\|_\infty < 1,\, \|\Delta_F\|_\infty < 1 \right\}$$

where $\Delta_F(\cdot)$ is a fictitious stable uncertainty element, which takes into account system performance requirements in the frequency domain (Fig. 8). The following robust performance theorem can be formulated as a generalization of the robust stability analysis theorem.

Theorem 2 (robust performance)[5]: For all $\Delta_P \in U(\Delta_{S_P})$, the closed-loop system shown in Fig. 8 is well posed, internally stable, and has robust performance if and only if

$$\sup_{\omega} \mu_{S_P}[M(j\omega)] \le 1$$

□

Because the definition of the function μ is not operative, one usually computes upper and lower bounds, for which efficient algorithms exist.[24]

B. μ Design of the SCAS for SCA

The most critical flight phases for a civil aircraft, from a maneuverability and controllability point of view, are the approach and landing; therefore a robust SCAS design has been carried out for the longitudinal axis flight control system, with reference to a classical flap on/landing gear down configuration, as shown in Fig. 1.

The closed-loop scheme, with the aircraft, the controller K, and the blocks associated to the uncertainty and the performance objectives, is shown in Fig. 9. The exogenous signals are pilot pitch angle command q_{cmd}, wind gust components in the longitudinal plane,

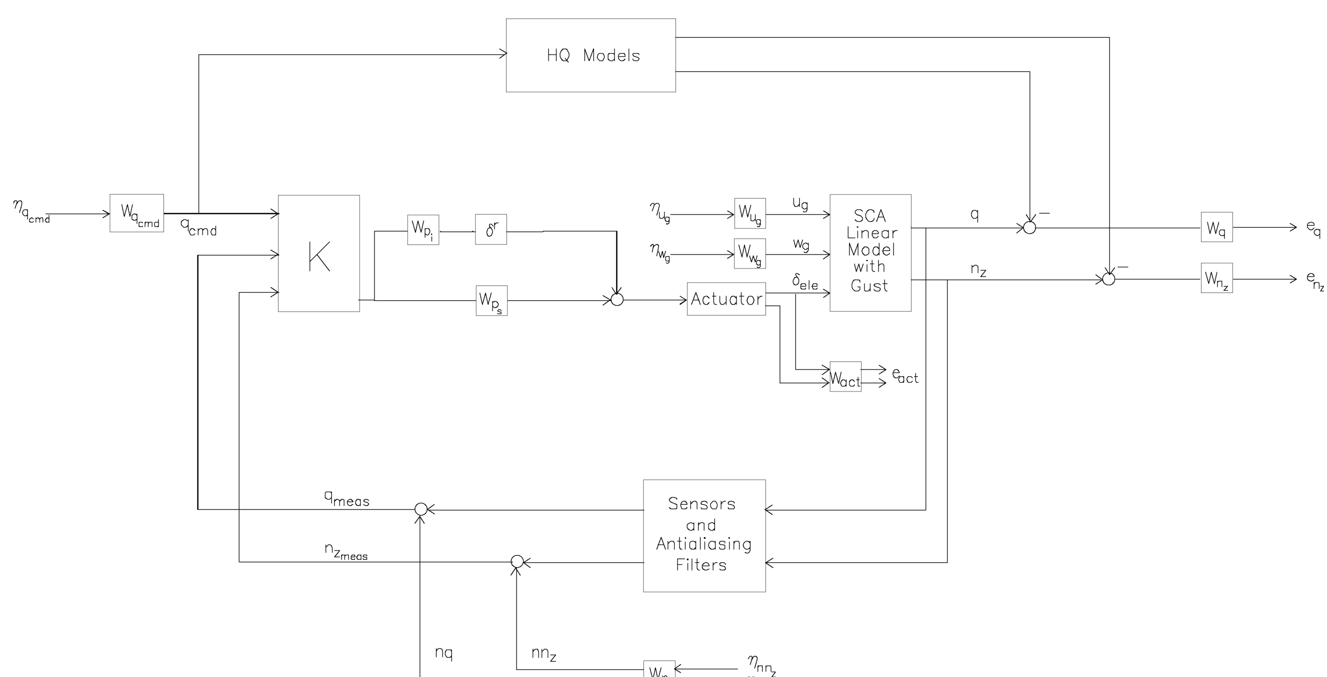

Fig. 9 SCA control system with weights.

u_g and w_g (Dryden modelization), and sensor noise (noise in the measured pitch angle n_q and normal acceleration n_{n_z}). According to the μ synthesis theory, the signals at the input of the closed-loop scheme of Fig. 9 have to be modeled as signals with unitary 2-norm, weighted in the frequency domain by suitable stable transfer functions (the so called weighting functions). In other words, if we denote the generic exogenous signal by h, it can be any member of the set

$$\left\{h = W_h \eta_h : \|\eta_h\|_2 \leq 1\right\}$$

where the stable weighting functions W_h allows the shaping of the spectrum of the signals entering the plant[5] and

$$\|\eta_h\|_2 = \sqrt{\int_0^{+\infty} \eta_h^T(t)\eta_h(t)\,\mathrm{d}t}$$

Moreover, the generalized error variables, that is, the signals that have to be kept small in spite of the exogenous signals and the plant uncertainty, are the outputs of the closed-loop scheme of Fig. 9. The errors we have considered are actuator signals limited in position and rate (elevator surface angular position and angular rate), that is, the vector $\boldsymbol{e}_{\mathrm{act}}$, and the pitch rate and the normal acceleration tracking errors for the model-matching problem, that is, the vectors $\boldsymbol{e}_q$ and $\boldsymbol{e}_{n_z}$.

Also, the error variables have to be weighted by frequency-dependent weights, to make them comparable and to reflect the performance objectives. For the closed-loop system of Fig. 9, the imposed MIMO performance objectives are as follows:

1) The aircraft response from the longitudinal stick to the pitch rate q and to the normal acceleration n_z should be as close as possible to the same response of two ideal models, which are chosen to satisfy FQ with level 1 requirements.

2) The elevator actuator has a maximum deflection of about 27 deg and a maximum slew rate of about 45 deg/s.

3) For the digital implementation of the controller, two low-pass, antialiasing filters at 75 Hz with unitary gains at dc are included for the q and n_z measures. Moreover, in the actuator model, a second-order Padé approximation of the time delay of $T = 0.0012$ s has been considered.

The relative weighting functions are chosen as follows:

1) W_{act} is a 2×2 diagonal matrix that allows the inclusion of the limits on actuator deflection and slew rate.

2) W_n is a 2×2 frequency-dependent diagonal matrix that provides frequency-domain models of sensor noise.

3) W_q and W_{n_z} are weight the tracking error signals. Because of the nonminimum phase characteristics of the model, two band-pass Butterworth filters have been chosen to reduce the error within the interval [0.05, 5] rad/s.

4) W_{u_g} and W_{w_g} are the Dryden filters used to model the typical experienced wind gusts velocity components.

5) $W_{q_{\mathrm{cmd}}}$ allows the shaping of the normalized stick input command to reproduce the expected pilot reference signal.

For this closed-loop weighted system, a robust performance control problem, with respect to a multiplicative, real, normalized plant input uncertainty δ_1^r, $|\delta_1^r| \leq 1$ has been solved. The actual description of the uncertainty is

$$\tfrac{1}{2} \leq \left|W_{P_i}\,\delta_1^r + W_{P_s}\right| \leq 3$$

to guarantee a prescribed stability margin, that is, 50% of input gain reduction and 200% of input gain increment, with $W_{P_i} = \frac{5}{4}$ and $W_{P_s} = \frac{7}{4}$. The structured uncertainty has been pulled out as shown in Fig. 6, and an LFT has been written for its characterization. Following the same notation of Sec. IV.A, in our particular case we have

$$m_r = 1, \qquad m_c = 0, \qquad m_C = 0 \rightarrow \boldsymbol{S} = (1)$$

Hence, for this uncertainty structure, we can define the set

$$\Delta_S := \left\{\Delta = \left(\delta_1^r\right)\right\}$$

and the corresponding uncertainty set $U(\Delta_S)$. The robust performance test requires the introduction of a fictitious uncertainty element Δ_F, and the augmentation of the uncertainty set Δ_S with Δ_{S_F}, $\Delta_{S_F} := \{\Delta_F = \Delta_1^C\}$, where $\Delta_1^C \in C^{5\times 4}$. In other words, Δ_F is a 5×4 full complex block that includes performance objectives.

According to the notation of Sec. VI.A., we have

$$\boldsymbol{S}_P = (1, 1), \qquad \Delta_P = \left\{\begin{bmatrix}\delta_1^r & 0\\ 0 & \Delta_1^C\end{bmatrix}, \delta_1^r \in \Delta_S, \Delta_1^C \in \Delta_{S_F}\right\}$$

At this point a controller guaranteeing robust performance is computed by using the so-called D–K iteration method[5−7] implemented in the MATLAB® toolbox.[24]

It will be shown in the next section that the designed controller covers a large region of the operating envelope, which includes the flap on/landing gear configuration shown in Fig. 1, by the ensuring of closed-loop longitudinal dynamic characteristics in accordance with level 1 FQ requirements and full equalization. To cover the whole flight envelope, the controller design described earlier has been repeated in a few points of the envelope, and a gain scheduled governing the transition between different regions has been implemented.

The final design step consists of integrating the robust CAS with existing classical module to implement automatic envelope protection and attitude-hold functionality. This has been done to allow, in agreement with the ADFCS project, pilot-in-the-loop simulations within a ground-based flight simulator. Even if off-line numerical evaluation shows that all analytical FQ requirements are satisfied, a favorable test pilot opinion is a necessary condition to consider an application of the proposed design procedure in the actual industrial world.

V. Off-Line Numerical Results

A first evaluation of the achieved results, both in terms of robustness and performance, has been obtained by means of numerical off-line simulations. These simulations have been performed via a specifically developed software environment that allows both linear and non-linear analysis of the augmented aircraft.

In Fig. 10, the pitch rate step responses achieved by using the designed robust SCAS are shown. To achieve analogous results by using a classical SISO FCL design approach, a heavy controller parameter scheduling would be required because the calibrated airspeed widely varies within the considered flight envelope region. In Fig. 11, we can see how the short-period characteristics change with the flight condition, if a fixed gain classical controller is used.

Moreover, from the FQ criteria evaluation, performed by using the LOES approach already described, we can see (Figs. 12 and 13) that the designed fixed gain controller guarantees level 1 FQ performance in the whole considered flight region even with significant variation of the calibrated airspeed.

In Table 1, the achieved stability margins obtained by breaking the loop at the elevator command level are listed. The requirement to have at least 6 dB of magnitude margin and 45 deg of phase margin is also satisfied.

In Fig. 14, the aircraft nonlinear responses corresponding to a small(+3 deg) pull to theta maneuver are shown.

In Fig. 15 a typical roll evaluation maneuver with stick centered is simulated to appreciate the achieved level of decoupling between

Table 1 SCAS gain and phase stability margin

FC	Altitude, ft	V_{cas}, kn	Alpha 1 g, deg	Gain margin, dB	Phase margin, deg
8	2000	140	4.49	17	91
9	5000	181	−0.2	10	49
10	2000	140	6.12	17	90
13	5000	151	2.85	14	73
14	2000	191	−0.91	9	46
17	3500	173	0.4	11	55
19	1500	128	6.87	19	105

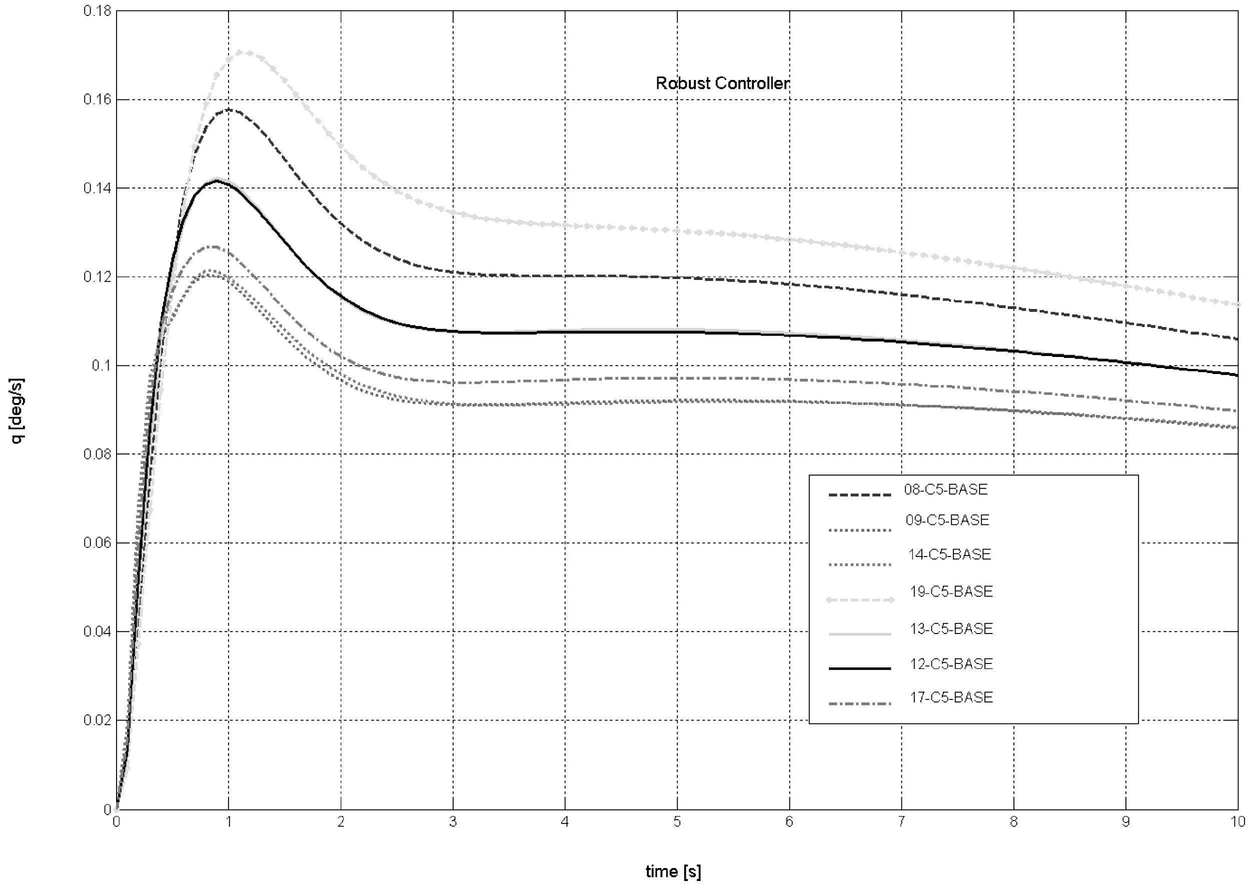

Fig. 10 Closed-loop linear step responses (robust SCAS).

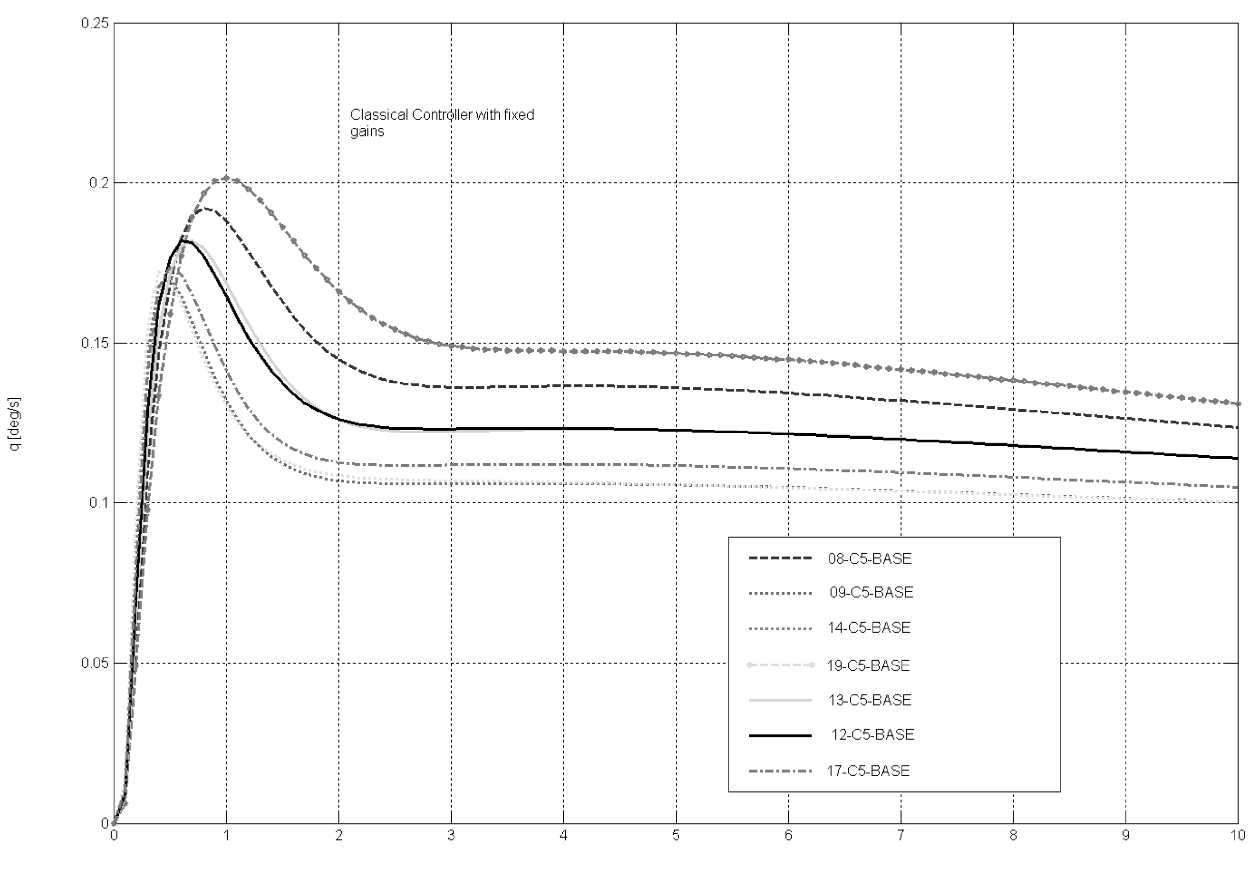

Fig. 11 Closed-loop linear step responses (classical SCAS).

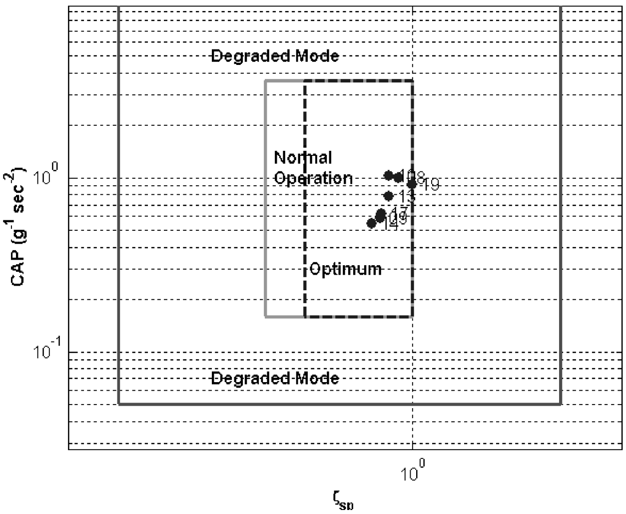

Fig. 12 CAP criterion.

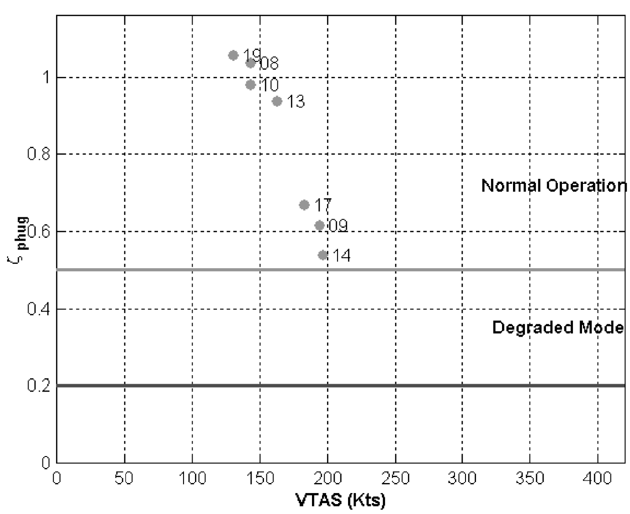

Fig. 13 Phugoid damping ratio criterion.

longitudinal and lateral axes, both at high and low airspeed. These results show an encouraging level of axis decoupling even at very low airspeed. With bank angle less than 20 deg, which represents a reasonable value for normal operations, the aircraft maintains altitude without requiring any pilot action along the longitudinal axis.

As mentioned, the design of the robust SCAS has been performed by taking into account simultaneous requirements on both the closed-loop dynamic characteristics and the rejection of external disturbances. Therefore, after the stability and performance analysis, we verified the response of the augmented aircraft to wind gust external disturbances. In particular, we evaluated the power spectral density of the pitch rate signals and the time histories of the pitch angle by means of a comparison with the results achieved by using classical FCL. The achieved results clearly show that the robust SCAS is much more insensitive to wind gust than the classical FCL (Fig. 16).

VI. Real-Time Simulator Results

Merely based on the achieved off-line numerical results, some significant potential benefits from the application of the μ synthesis theory to FCL design have been identified. However, a fundamental step for a future use of the proposed FCL design technique in a real industrial application is to evaluate the opinions of the pilots as end users of the system. In other words, it is necessary to answer the following question: Is the aircraft behavior acceptable for the pilots? To this end, within the framework of the ADFCS research project, a specific simulator evaluation session was performed to verify, by means of pilot-in-the-loop simulations, if the proposed robust FCL can guarantee at least the same FQ characteristics achieved by using classical FCL.

A. NLR Simulator Facility

The pilot-in-the-loop simulation tests have been conducted by using the NLR Research Flight Simulator (RFS). The RFS is based on a (four-degree-of-freedom [DOF]) motion platform (Fig. 17) and distinguishes itself from training simulators by its great flexibility to adapt to all kinds of research projects. For this purpose, the design strategy applied has a high level of modularity, with interchangeable hardware, as well as software components.

When it is taken into account that the SCA model involved is a twin jet aircraft, for the cockpit it has been decided to use the Fokker-100 layout (Fig. 18). The cockpit has a full glass configuration with

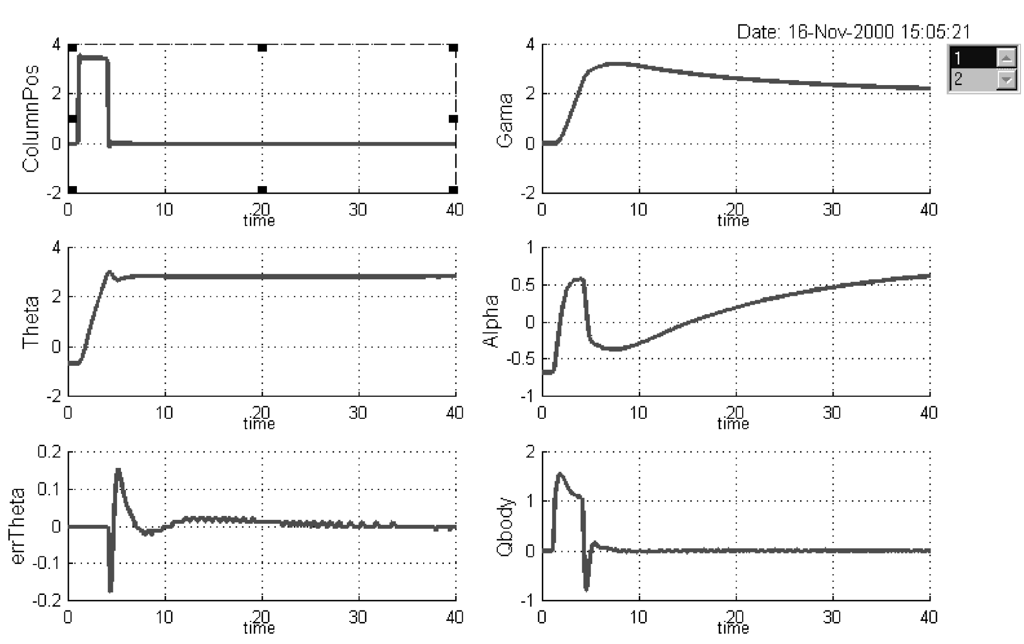

Fig. 14 Pull to theta (+3 deg), 180 kn.

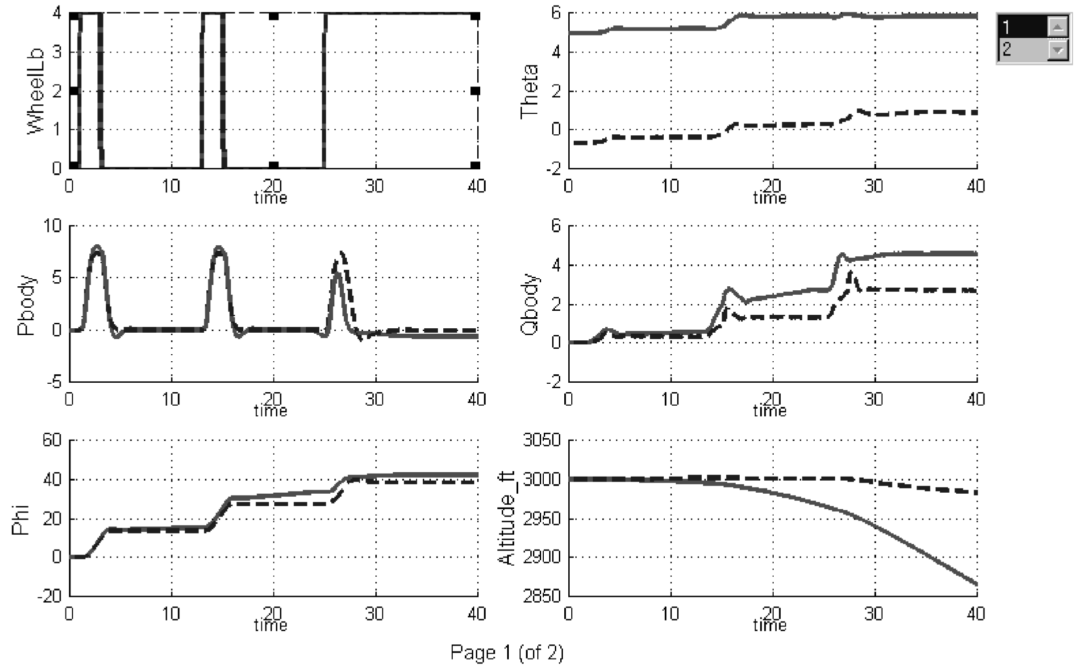

Fig. 15 Roll evaluation, 140/180 kn: ——, low speed and – – –, high speed.

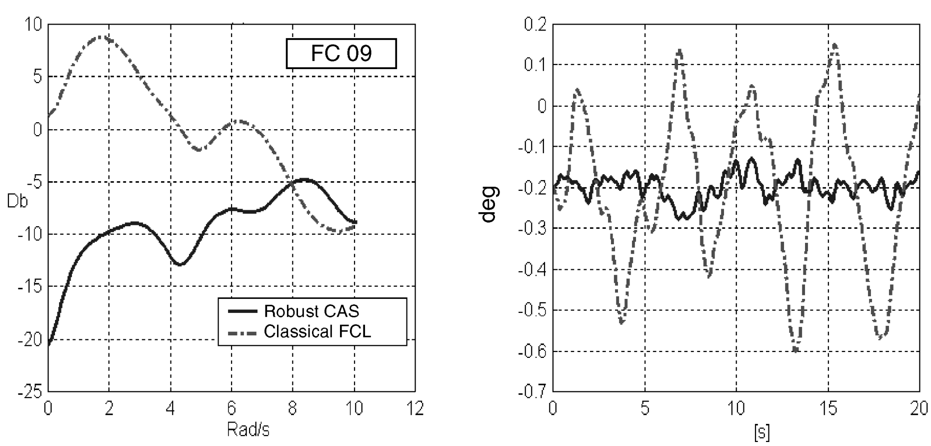

Fig. 16 Gust response, robust/classical FCL.

Fig. 17 NLR RFS, four-DOF motion base.

Fig. 18 NLR RFS cockpit.

an out of window view of the Amsterdam surrounding the Schipol airport runway.

B. Test Scenario

The tested robust FCL configuration consists of the fixed gain longitudinal SCAS, designed by use of a robust model matching approach based on the μ synthesis theory, integrated with the classical lateral controller and envelope protection modules. As already specified, the designed robust control is intended to work in the approach and landing configuration. The maneuvers, executed at various points of the considered flight envelope, are summarized in the following: 1) general flight from trimmed initial conditions; 2) open-loop exercises with a) small singlets in pitch, b) pull/push to θ (± 10 deg) and pull/push and release column to stabilize on a predetermined value of θ; and c) turn entry from straight and level flight ($\phi = 15, 30, 45$ deg); 3) acceleration and deceleration with wings level; 4) slats/flaps extension (to fully extended position) and retraction; 5) engine steps, from trim throttle to idle/full position; and 6) final approach and landing.

C. Pilot Evaluation Results

The achieved results, expressed in terms of pilots' opinions, can be summarized as follows:

The pitch control during maneuvers results to be very stable (level 1) in the whole considered flight region. This result was achieved by a fixed gain controller without any scheduling with respect to V_{cas} as required in the classical control laws.

To maintain the desired pitch attitude with adequate pilot workload, the activation of the theta-hold functionality was required. Note that the use of theta hold is not typical for a conventional

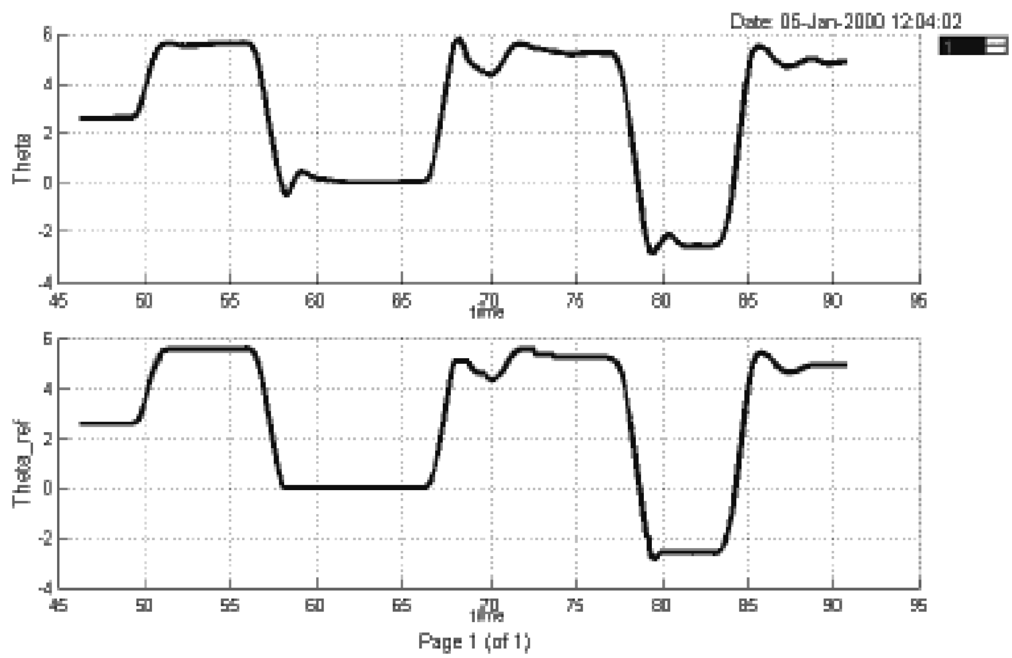

Fig. 19 Simulator results: pull push to theta.

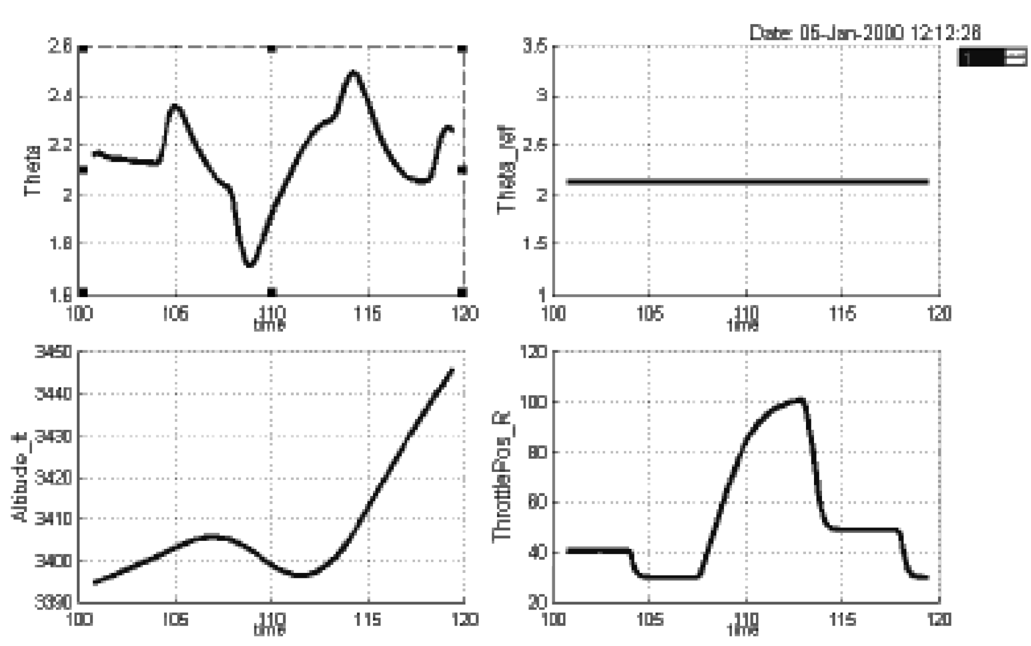

Fig. 20 Simulator results: engine steps.

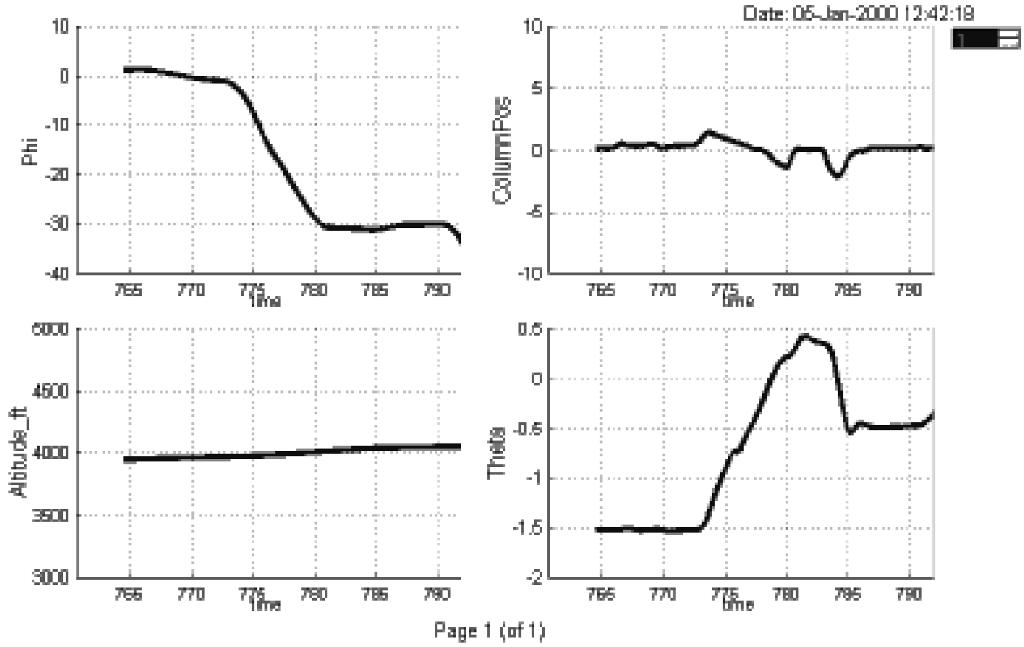

Fig. 21 Simulator results: roll evaluation.

aircraft; indeed it derives from a specific pilots' requirement (for the closed loop FBW flight control system) of attitude hold with stick centered.

The designed theta-hold system guarantees good performance during transients due to engines and configuration changes and in gross push/pull to theta maneuvers (Figs. 19 and 20). Synthetically, it is rated level 1.

On the other hand, the pilots noted the difficulty (level 2) in small theta changes (0.5–1 deg). The optimization of the switching logic used to control the activation of the theta-old function may solve this problem.

As in the classical case, landing with the pitch rate demand control law is difficult (level 2–3) in the flare maneuver. (It overshoots the desired pitch attitude and requires a stick push from the pilot that is not natural.) The integration of the same α-feedback module used in the classical control laws approach has been also tested. A conflict with the theta-hold function has been recognized and needs to be better investigated.

The sensitivity of the pitch control behavior with respect to the center of gravity and weight variation has been evaluated. The conclusion of the pilots was that good FQ along the pitch axis are guaranteed in the whole weight and balance envelope. The same result has been achieved by use of the classical FCL.

Very good decoupling between lateral and longitudinal axis is achieved during sustained turn maneuvers (Fig. 21) that was rated to be level 1 by the pilots.

VII. Conclusions

In this paper the application of a robust control technique based on the μ synthesis theory to a realistic flight control benchmark problem for a SCA has been considered.

It has been shown that the systematic approach to the FCL design problem behind modern techniques can highly simplify the design cycle and make it more transparent. To this end, an effort has been made to formulate a suitable set of FQ requirements for a SCA and to take into account such requirements during the control laws design process by choosing an ad hoc reference dynamic aircraft model.

In particular, the application of the design methodology under investigation removes the time-consuming classical one-loop-at-time approach, reduces the number of design points for which a controller has to be tuned, and allows automated software tools to be used for control laws synthesis. Also, multivariable performance and robustness requirements can be taken into account during the design phase, which renders this approach suitable for new-generation aircraft employing multiple coupled control loops.

Another important point is that the designed controller has been validated through both off-line simulations and pilot-in-the-loop simulator tests. The successful simulator evaluation is a significant result of the project that represented one of the first opportunities in Europe to evaluate the potential benefits of the μ analysis and synthesis theory by means of pilot-in-the-loop simulations.

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